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$$\therefore \frac{n(n+1)(n+2)\dots\dots(n+m-1)}{m!} - n\frac{n(n+1)\dots(n+m-4)}{(m-3)!}$$

$$+\frac{n(n-1)}{1.2.3}\frac{n(n+1)\dots(n+m-7)}{(m-6)!}\dots$$

$$=\frac{n!}{(n-m)!\,m!}+\frac{n!}{(n-m+1)!\,(m-2)!\,1!}+\frac{n!}{(n-m+2)!\,(m-4)!\,2!}+\dots$$

$$+\frac{n!}{(2n-m)!\,(m-2n)!\,n!}.$$

Now, if $m>2n$, there are no terms in the expansion of $(1+x+x^2)^n$ of which the exponent of x is m. Hence, the second member of the above equation is zero. When $m=2n$, there exists only one term of which the exponent of x is $m=2n$, and its coefficient is the last term of the above equation, which is equal to 1.

The fact that there are no terms of which the exponent of x is m when $m>2n$ is manifested also by some of the signs in the factors of the denominators becoming negative.

359. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Illinois.

Show when $1/(1-x)(1-x^3)(1-x^5)(1-x^7)\dots=(1+x)(1+x^2)(1+x^3)(1+x^4)\dots$

Solution by B. F. FINKEL, Ph. D., Drury College.

$$\frac{1}{(1-x)(1-x^3)(1-x^5)\dots(1-x^{2n-1})}$$

$$\equiv\frac{(1+x)(1+x^2)(1+x^3)\dots(1+x^n)}{(1-x)(1+x)(1+x^2)(1-x^3)(1+x^3)(1+x^4)\dots(1-x^{2n-1})}$$

$$\equiv\frac{(1+x)(1+x^2)(1+x^3)\dots(1+x^n)}{(1-x^{2^{p-1}})(1-x^{3.2^{q-1}})(1-x^{5.2^{r-1}})(1-x^{7.2^{s-1}})\dots\dots(1-x^{2n-1})},$$

where $p+q+r+s+\dots=4n-4$,

$$\doteq(1+x)(1+x^2)(1+x^3)\dots(1+x^n)\dots \text{ as } n=\infty,$$

when $-1<x<1$, since, when $-1<x<1$, each factor in the denominator approaches 1.

Note. The editors desire that contributors send in good problems for solution. Let us have a great variety of problems for solutions in the various departments. Also send in solutions; we prefer to publish solutions prepared by contributors, rather than publish our own. We have neither time, inclination, nor ability to solve every problem proposed in the **Monthly**. However, if every contributor will give a little time to the problems each month, by united effort, there will be few problems remain unsolved. We have recently republished a number of unsolved problems, and we shall be pleased to have solutions of any of them, or any others that remain unsolved. **Ed, F.**

GEOMETRY.

384. Proposed by S. LEFSHETZ, Universityof Nebraska.

Let ABC be a triangle, O a circle tangent to its three sides, T a variable tangent of O, which cuts the sides BC, CA, AB in a, b, c. Oa', Ob', Oc' the perpendiculars in O to Oa, Ob, Oc, cutting, respectively, T in points a', b', c'. Prove that Aa', Bb', Cc' meet in a point t, and find the locus of t when T varies. Purely geometrical proofs wanted.

Solution by R. P. BAKER, Iowa City, Iowa.

I. By elementary geometry.

Lemma I. The second tangent from a', $a'K$ to the circle is parallel to BC.

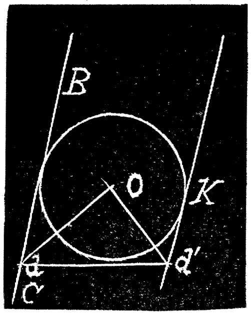

Fig. 1.

For $a'aB$, $aa'A$ are the doubles of the complementary angles $a'aO$ and $aa'O$, and hence supplementary. So $a'K$ is parallel to aB, that is to CB.

Lemma II. If the tangents to a circle from P meet two parallel tangents in Q, Q'; R, R', respectively, and O is the center; then $PQ.PR=OP^2=PQ'.PR'$.

Let M, N be the points of contact of PQ, PR, and S, T of the parallel tangents.

Then $OPQ=OPR$; $2OQP=$supp. $SOM=$supp. $(SOP-MOP)=$supp. $SOP+MOP$.

$2POR=2POT-TON=2POT-(POT-NOP)=POT+MOP=$supp. $SOP+MOP$. Therefore, $OQP=POR$, and the triangles OPQ and RPO are similar.

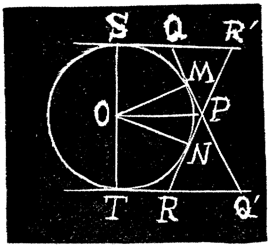

Fig. 2.

Hence $PQ.PR=OP^2$, and by the similar triangles PRQ', $PR'Q$, each is equal to $PQ'.PR'$.

Applying the lemmas to the figure, we have $ab'.aC=ac'.aB$, and the triangles aBb', aCc' are similar, having equal angles at a, and Bb', Cc' are parallel. So for Aa', Bb'.

II. By Brianchon's Theorem.

If the second tangents from b', c' meet at A', we have in AB, BC, CA, $b'A'$, bc, $c'A'$ six tangents to a conic. If the lines are taken in the order written the joins of the cuts of opposite pairs are Bb', Cc', and the line at infinity. Hence Bb', Cc' are parallel, and similarly, the other pairs.